

# Package ‘SPCAvRP’

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**Type** Package

**Title** Sparse Principal Component Analysis via Random Projections  
(SPCAvRP)

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**Description** Implements the SPCA<sub>v</sub>RP algorithm, developed and analysed in "Sparse principal component analysis via random projections" Gataric, M., Wang, T. and Samworth, R. J. (2018) <[arXiv:1712.05630](https://arxiv.org/abs/1712.05630)>. The algorithm is based on the aggregation of eigenvector information from carefully-selected random projections of the sample covariance matrix.

**Depends** R (>= 3.0.0), parallel, MASS

**License** GPL-3

**URL** <https://arxiv.org/abs/1712.05630>

**NeedsCompilation** no

**Repository** CRAN

**RoxygenNote** 6.1.1

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SPCAvRP

*Computes the leading eigenvector using the SPCAvRP algorithm***Description**

Computes  $l$ -sparse leading eigenvector of the sample covariance matrix, using  $A \times B$  random axis-aligned projections of dimension  $d$ . For the multiple component estimation use [SPCAvRP\\_subspace](#) or [SPCAvRP\\_deflation](#).

**Usage**

```
SPCAvRP(data, cov = FALSE, l, d = 20, A = 600, B = 200,
center_data = TRUE, parallel = FALSE,
cluster_type = "PSOCK", cores = 1, machine_names = NULL)
```

**Arguments**

<code>data</code>	Either the data matrix ( $p \times n$ ) or the sample covariance matrix ( $p \times p$ ).
<code>cov</code>	TRUE if data is given as a sample covariance matrix.
<code>l</code>	Desired sparsity level in the final estimator (see Details).
<code>d</code>	The dimension of the random projections (see Details).
<code>A</code>	Number of projections over which to aggregate (see Details).
<code>B</code>	Number of projections in a group from which to select (see Details).
<code>center_data</code>	TRUE if the data matrix should be centered (see Details).
<code>parallel</code>	TRUE if the selection step should be computed in parallel by uses package "parallel".
<code>cluster_type</code>	If <code>parallel == TRUE</code> , this can be "PSOCK" or "FORK" (cf. package "parallel").
<code>cores</code>	If <code>parallel == TRUE</code> and <code>cluster_type == "FORK"</code> , number of cores to use.
<code>machine_names</code>	If <code>parallel == TRUE</code> , the names of the computers on the network.

**Details**

This function implements the SPCAvRP algorithm for the principal component estimation (Algorithm 1 in the reference given below).

If the true sparsity level  $k$  is known, use  $l = k$  and  $d = k$ .

If the true sparsity level  $k$  is unknown,  $l$  can take an array of different values and then the estimators of the corresponding sparsity levels are computed. The final choice of  $l$  can then be done by the user via inspecting the explained variance computed in the output `value` or via inspecting the output `importance_scores`. The default choice for  $d$  is 20, but we suggest choosing  $d$  equal to or slightly larger than  $l$ .

It is desirable to choose  $A$  (and  $B = \text{ceiling}(A/3)$ ) as big as possible subject to the computational budget. In general, we suggest using  $A = 300$  and  $B = 100$  when the dimension of data is a few hundreds, while  $A = 600$  and  $B = 200$  when the dimension is on order of 1000.

If `center_data == TRUE` and `data` is given as a data matrix, the first step is to center it by executing `scale(data, center_data, FALSE)`, which subtracts the column means of `data` from their corresponding columns.

If `parallel == TRUE`, the parallelised SPCA<sub>v</sub>RP algorithm is used. We recommend to use this option if `p`, `A` and `B` are very large.

### Value

Returns a list of three elements:

<code>vector</code>	A matrix of dimension $p \times \text{length}(1)$ with columns as the estimated eigenvectors of sparsity level <code>l</code> .
<code>value</code>	An array with $\text{length}(1)$ eigenvalues corresponding to the estimated eigenvectors returned in <code>vector</code> .
<code>importance_scores</code>	An array of length <code>p</code> with importance scores for each variable <code>1</code> to <code>p</code> .

### Author(s)

Milana Gataric, Tengyao Wang and Richard J. Samworth

### References

Milana Gataric, Tengyao Wang and Richard J. Samworth (2018) Sparse principal component analysis via random projections <https://arxiv.org/abs/1712.05630>

### Examples

```
p <- 100 # data dimension
k <- 10 # true sparsity level
n <- 1000 # number of observations
v1 <- c(rep(1/sqrt(k), k), rep(0,p-k)) # true principal component
Sigma <- 2*tcrossprod(v1) + diag(p) # population covariance
mu <- rep(0, p) # population mean
loss = function(u,v){
  # the loss function
  sqrt(abs(1-sum(v*u)^2))
}
set.seed(1)
X <- mvrnorm(n, mu, Sigma) # data matrix

spcavrp <- SPCAvRP(data = X, cov = FALSE, l = k, d = k, A = 200, B = 70)
spcavrp.loss <- loss(v1,spcavrp$vector)
print(paste0("estimation loss when l=d=k=10, A=200, B=70: ", spcavrp.loss))

##choosing sparsity level l if k unknown:
#spcavrp.choosel <- SPCAvRP(data = X, cov = FALSE, l = c(1:30), d = 15, A = 200, B = 70)
#plot(1:p,spcavrp.choosel$importance_scores,xlab='variable',ylab='w',
# main='choosing l when k unknown: \n importance scores w')
#plot(1:30,spcavrp.choosel$value,xlab='l',ylab='Var_l',
# main='choosing l when k unknown: \n explained variance Var_l')
```

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SPCAvRP_deflation	<i>Computes multiple principal components using our modified deflation scheme</i>
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### Description

Computes  $m$  leading eigenvectors of the sample covariance matrix which are sparse and orthogonal, using the modified deflation scheme in conjunction with the SPCA<sub>v</sub>RP algorithm.

### Usage

```
SPCAvRP_deflation(data, cov = FALSE, m, l, d = 20,
  A = 600, B = 200, center_data = TRUE)
```

### Arguments

<code>data</code>	Either the data matrix ( $p \times n$ ) or the sample covariance matrix ( $p \times p$ ).
<code>cov</code>	TRUE if data is given as a sample covariance matrix.
<code>m</code>	The number of principal components to estimate.
<code>l</code>	The array of length $m$ with the desired sparsity of $m$ principle components (see Details).
<code>d</code>	The dimension of the random projections (see Details).
<code>A</code>	Number of projections over which to aggregate (see Details).
<code>B</code>	Number of projections in a group from which to select (see Details).
<code>center_data</code>	TRUE if the data matrix should be centered (see Details).

### Details

This function implements the modified deflation scheme in conjunction with SPCA<sub>v</sub>RP (Algorithm 2 in the reference given below).

If the true sparsity level is known and for each component is equal to  $k$ , use  $d = k$  and  $l = \text{rep}(k, m)$ . Sparsity levels of different components may take different values. If  $k$  is unknown, appropriate  $k$  could be chosen from an array of different values by inspecting the explained variance for one component at the time and by using SPCA<sub>v</sub>RP in a combination with the deflation scheme implemented in SPCA<sub>v</sub>RP\_deflation.

It is desirable to choose  $A$  (and  $B = \text{ceiling}(A/3)$ ) as big as possible subject to the computational budget. In general, we suggest using  $A = 300$  and  $B = 100$  when the dimension of data is a few hundreds, while  $A = 600$  and  $B = 200$  when the dimension is on order of 1000.

If `center_data == TRUE` and `data` is given as a data matrix, the first step is to center it by executing `scale(data, center_data, FALSE)`, which subtracts the column means of `data` from their corresponding columns.

**Value**

Returns a list of two elements:

vector            A matrix whose  $m$  columns are the estimated eigenvectors.  
 value            An array with  $m$  estimated eigenvalues.

**Author(s)**

Milana Gataric, Tengyao Wang and Richard J. Samworth

**References**

Milana Gataric, Tengyao Wang and Richard J. Samworth (2018) Sparse principal component analysis via random projections <https://arxiv.org/abs/1712.05630>

**See Also**

[SPCAvRP](#), [SPCAvRP\\_subspace](#)

**Examples**

```
p <- 50 # data dimension
k <- 8 # true sparsity of each component
v1 <- 1/sqrt(k)*c(rep(1, k), rep(0, p-k)) # first principal component (PC)
v2 <- 1/sqrt(k)*c(rep(0,4), 1, -1, 1, -1, rep(1,4), rep(0,p-12)) # 2nd PC
v3 <- 1/sqrt(k)*c(rep(0,6), 1, -rep(1,4), rep(1,3), rep(0,p-14)) # 3rd PC
Sigma <- diag(p) + 40*tcrossprod(v1) + 20*tcrossprod(v2) + 5*tcrossprod(v3) # population covariance
mu <- rep(0, p) # population mean
n <- 2000 # number of observations
loss = function(u,v){
  sqrt(abs(1-sum(v*u)^2))
}
loss_sub = function(U,V){
  U<-qr.Q(qr(U)); V<-qr.Q(qr(V))
  norm(tcrossprod(U)-tcrossprod(V),"2")
}
set.seed(1)
X <- mvrnorm(n, mu, Sigma) # data matrix

spcavrp.def <- SPCAvRP_deflation(data = X, cov = FALSE, m = 2, l = rep(k,2),
                               d = k, A = 200, B = 70, center_data = FALSE)
subspace_estimation<-data.frame(
  loss_sub(matrix(c(v1,v2),ncol=2),spcavrp.def$vector),
  loss(spcavrp.def$vector[,1],v1),
  loss(spcavrp.def$vector[,2],v2),
  crossprod(spcavrp.def$vector[,1],spcavrp.def$vector[,2]))
colnames(subspace_estimation)<-c("loss_sub","loss_v1","loss_v2","inner_prod")
rownames(subspace_estimation)<-c("")
print(subspace_estimation)
```

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SPCAvRP_subspace	<i>Computes the leading eigenspace using the SPCA<sub>v</sub>RP algorithm for the eigenspace estimation</i>
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### Description

Computes  $m$  leading eigenvectors of the sample covariance matrix which are sparse and orthogonal, using  $A \times B$  random axis-aligned projections of dimension  $d$ .

### Usage

```
SPCAvRP_subspace(data, cov = FALSE, m, l, d = 20,
  A = 600, B = 200, center_data = TRUE)
```

### Arguments

data	Either the data matrix ( $p \times n$ ) or the sample covariance matrix ( $p \times p$ ).
cov	TRUE if data is given as a sample covariance matrix.
m	The dimension of the eigenspace, i.e the number of principal components to compute.
l	Desired sparsity level of the eigenspace (i.e. the number of non-zero rows in output\$vector) (see Details).
d	The dimension of the random projections (see Details).
A	Number of projections over which to aggregate (see Details).
B	Number of projections in a group from which to select (see Details).
center_data	TRUE if the data matrix should be centered (see Details).

### Details

This function implements the SPCA<sub>v</sub>RP algorithm for the eigenspace estimation (Algorithm 3 in the reference given below).

If the true sparsity level  $k$  of the eigenspace is known, use  $l = k$  and  $d = k$ .

If the true sparsity level  $k$  of the eigenspace is unknown, the appropriate choice of  $l$  can be done, for example, by running the algorithm (for any  $l$ ) and inspecting the computed output `importance_scores`. The default choice for  $d$  is 20, but we suggest choosing  $d$  equal to or slightly larger than  $l$ .

It is desirable to choose  $A$  (and  $B = \text{ceiling}(A/3)$ ) as big as possible subject to the computational budget. In general, we suggest using  $A = 300$  and  $B = 100$  when the dimension of data is a few hundreds, while  $A = 600$  and  $B = 200$  when the dimension is on order of 1000.

If `center_data == TRUE` and data is given as a data matrix, the first step is to center it by executing `scale(data, center_data, FALSE)`, which subtracts the column means of data from their corresponding columns.

**Value**

Returns a list of two elements:

`vector`            A matrix whose  $m$  columns are the estimated eigenvectors.  
`value`             An array with  $m$  estimated eigenvalues.  
`importance_scores`  
                     An array of length  $p$  with importance scores for each variable 1 to  $p$ .

**Author(s)**

Milana Gataric, Tengyao Wang and Richard J. Samworth

**References**

Milana Gataric, Tengyao Wang and Richard J. Samworth (2018) Sparse principal component analysis via random projections <https://arxiv.org/abs/1712.05630>

**See Also**

[SPCAvRP](#), [SPCAvRP\\_deflation](#)

**Examples**

```
p <- 50 # data dimension
k1 <- 8 # sparsity of each individual component
v1 <- 1/sqrt(k1)*c(rep(1, k1), rep(0, p-k1)) # first principal component (PC)
v2 <- 1/sqrt(k1)*c(rep(0,4), 1, -1, 1, -1, rep(1,4), rep(0,p-12)) # 2nd PC
v3 <- 1/sqrt(k1)*c(rep(0,6), 1, -rep(1,4), rep(1,3), rep(0,p-14)) # 3rd PC
Sigma <- diag(p) + 40*tcrossprod(v1) + 20*tcrossprod(v2) + 5*tcrossprod(v3) # population covariance
mu <- rep(0, p) # population mean
n <- 2000 # number of observations
loss = function(u,v){
  sqrt(abs(1-sum(v*u)^2))
}
loss_sub = function(U,V){
  U<-qr.Q(qr(U)); V<-qr.Q(qr(V))
  norm(tcrossprod(U)-tcrossprod(V),"2")
}
set.seed(1)
X <- mvrnorm(n, mu, Sigma) # data matrix

spcavrp.sub <- SPCAvRP_subspace(data = X, cov = FALSE, m = 2, l = 12, d = 12,
                              A = 200, B = 70, center_data = FALSE)

subspace_estimation<-data.frame(
  loss_sub(matrix(c(v1,v2),ncol=2),spcavrp.sub$vector),
  loss(spcavrp.sub$vector[,1],v1),
  loss(spcavrp.sub$vector[,2],v2),
  crossprod(spcavrp.sub$vector[,1],spcavrp.sub$vector[,2]))
colnames(subspace_estimation)<-c("loss_sub","loss_v1","loss_v2","inner_prod")
```

```
rownames(subspace_estimation)<-c("")  
print(subspace_estimation)  
  
plot(1:p,spcavrp.sub$importance_scores,xlab='variable',ylab='w',  
     main='importance scores w \n (may use to choose l when k unknown)')
```



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