Package 'SuperGauss'

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Description
Likelihood evaluations for stationary Gaussian time series are typically obtained via the Durbin-Levinson algorithm, which scales as O(n^2) in the number of time series observations. This package provides a ``superfast'' O(n log^2 n) algorithm written in C++, crossing over with Durbin-Levinson around n = 300. Efficient implementations of the score and Hessian functions are also provided, leading to superfast versions of inference algorithms such as Newton-Raphson and Hamiltonian Monte Carlo. The C++ code provides a Toeplitz matrix class packaged as a header-only library, to simplify low-level usage in other packages and outside of R.

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`SuperGauss-package` *Superfast inference for stationary Gaussian time series.*

Description

Likelihood evaluations for stationary Gaussian time series are typically obtained via the Durbin-Levinson algorithm, which scales as O(n^2) in the number of time series observations. This package provides a "superfast" O(n log^2 n) algorithm written in C++, crossing over with Durbin-Levinson around n = 300. Efficient implementations of the score and Hessian functions are also provided, leading to superfast versions of inference algorithms such as Newton-Raphson and Hamiltonian Monte Carlo. The C++ code provides a Toeplitz matrix class packaged as a header-only library, to simplify low-level usage in other packages and outside of R.

Details

While likelihood calculations with stationary Gaussian time series generally scale as `O(N^2)` in the number of observations, this package implements an algorithm which scales as `O(N log^2 N)`. "Superfast" algorithms for loglikelihood gradients and Hessians are also provided. The underlying C++ code is distributed through a header-only library found in the installed package's `include` directory.

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Examples

```
# Superfast inference for the timescale parameter
# of the exponential autocorrelation function
exp_acf <- function(lambda) exp(-(1:N-1)/lambda)

# simulate data
lambda0 <- 1
N <- 1000
X <- rnormtz(n = 1, acf = exp_acf(lambda0))

# loglikelihood function
# allocate memory for a NormalToeplitz distribution object
NTz <- NormalToeplitz$new(N)
loglik <- function(lambda) {
  NTz$logdens(z = X, acf = exp_acf(lambda))
  ## dSnorm(X = X, acf = Toep, log = TRUE)
}

# maximum likelihood estimation
optimize(f = loglik, interval = c(.2, 5), maximum = TRUE)
```

---

acf2incr — *Convert position autocorrelations to increment autocorrelations.*

Description

Convert the autocorrelation of a stationary sequence `x = (x_1, ..., x_N)` to that of its increments, `dx = (x_2 - x_1, ..., x_N - x_(N-1))`.

Usage

```
acf2incr(acf)
```

Arguments

acf	Length-N vector of position autocorrelations.

Value

Length N-1 vector of increment autocorrelations.

Examples

```
acf2incr(acf = exp(-(0:10)))
```

acf2msd — *Convert autocorrelation of stationary increments to mean squared displacement of posititions.*

Description

Converts the autocorrelation of a stationary increment sequence `dx = (x_1 - x_0, ..., x_N - x_(N-1))` to the mean squared displacement (MSD) of the corresponding positions, i.e., `MSD_i = E[(x_i - x_0)^2]`.

Usage

```
acf2msd(acf)
```

Arguments

acf	Length-`N` autocorrelation vector of a stationary increment sequence.

Value

Length-`N` MSD vector of the corresponding positions.

Examples

```
acf2msd(acf = exp(-(0:10)))
```

Cholesky — *Cholesky multiplication with Toeplitz variance matrices.*

Description

Multiplies the Cholesky decomposition of the Toeplitz matrix with another matrix, or solves a system of equations with the Cholesky factor.

Usage

```
cholZX(Z, acf)

cholXZ(X, acf)
```

Arguments

Z	Length-`N` or `N x p` matrix of residuals.
acf	Length-`N` autocorrelation vector of the Toeplitz variance matrix.
X	Length-`N` or `N x p` matrix of observations.

Details

If `C == t(chol(toeplitz(acf)))`, then `cholZX()` computes `C %*% Z` and `cholZX()` computes `solve(C, X)`. Both functions use the Durbin-Levinson algorithm.

Value

Size `N x p` residual or observation matrix.

Examples

```
N <- 10
p <- 2
acf <- exp(-(1:N - 1))

Z <- matrix(rnorm(N * p), N, p)
cholZX(Z = Z, acf = acf) - (t(chol(toeplitz(acf))) %*%  Z)

X <- matrix(rnorm(N * p), N, p)
cholXZ(X = X, acf = acf) - solve(t(chol(toeplitz(acf))), X)
```

`Circulant` *Constructor and methods for Circulant matrix objects.*

Description

Constructor and methods for Circulant matrix objects.

Methods

Public methods:

- `Circulant$new()`
- `Circulant$size()`
- `Circulant$set_acf()`
- `Circulant$get_acf()`
- `Circulant$set_psd()`
- `Circulant$get_psd()`
- `Circulant$has_acf()`
- `Circulant$prod()`
- `Circulant$solve()`
- `Circulant$log_det()`
- `Circulant$clone()`

Method `new()`**:** Class constructor.

Usage:

```
Circulant$new(N, uacf, upsd)
```

Arguments:

`N` Size of Circulant matrix.

`uacf` Optional vector of `Nu = floor(N/2)+1` unique elements of the autocorrelation.

`upsd` Optional vector of `Nu = floor(N/2)+1` unique elements of the PSD.

Returns: A `Circulant` object.

Method `size()`**:** Get the size of the Circulant matrix.

Usage:

```
Circulant$size()
```

Returns: Size of the Circulant matrix.

Method `set_acf()`**:** Set the autocorrelation of the Circulant matrix.

Usage:

```
Circulant$set_acf(uacf)
```

Arguments:

`uacf` Vector of `Nu = floor(N/2)+1` unique elements of the autocorrelation.

Method `get_acf()`**:** Get the autocorrelation of the Circulant matrix.

Usage:

```
Circulant$get_acf()
```

Returns: The complete autocorrelation vector of length `N`.

Method `set_psd()`**:** Set the PSD of the Circulant matrix.

The power spectral density (PSD) of a Circulant matrix `Ct = Circulant(acf)` is defined as `psd = iFFT(acf)`.

Usage:

```
Circulant$set_psd(upsd)
```

Arguments:

`upsd` Vector of `Nu = floor(N/2)+1` unique elements of the psd.

Method `get_psd()`**:** Get the PSD of the Circulant matrix.

Usage:

```
Circulant$get_psd()
```

Returns: The complete PSD vector of length `N`.

Method `has_acf()`**:** Check whether the autocorrelation of the Circulant matrix has been set.

Usage:

```
Circulant$has_acf()
```

Returns: Logical; `TRUE` if `Circulant$set_acf()` has been called.

Method `prod()`**:** Circulant matrix-matrix product.

Usage:

```
Circulant$prod(x)
```

Arguments:

`x` Vector or matrix with `N` rows.

Returns: The matrix product `Ct %*% x`.

Method `solve()`**:** Solve a Circulant system of equations.

Usage:

```
Circulant$solve(x)
```

Arguments:

`x` Optional vector or matrix with `N` rows.

Returns: The solution in `z` to the system of equations `Ct %*% z = x`. If `x` is missing, returns the inverse of `Ct`.

Method `log_det()`**:** Calculate the log-determinant of the Circulant matrix.

Usage:

```
Circulant$log_det()
```

Returns: The log-determinant `log(det(Ct))`.

Method `clone()`**:** The objects of this class are cloneable with this method.

Usage:

```
Circulant$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

`dnormtz` *Density of a multivariate normal with Toeplitz variance matrix.*

Description

Density of a multivariate normal with Toeplitz variance matrix.

Usage

```
dnormtz(X, mu, acf, log = FALSE, method = c("gschur", "ltz"))
```

Arguments

`X`	Vector of length `N` or `N x n` matrix, of which each column is a multivariate observation.
`mu`	Vector or matrix of mean values of compatible dimensions with `X`. Defaults to all zeros.
`acf`	Vector of length `N` containing the first column of the Toeplitz variance matrix.
`log`	Logical; whether to return the multivariate normal density on the log scale.
`method`	Which calculation method to use. Choices are: `gschur` for a modified version of the Generalized Schur algorithm of Ammar & Gragg (1988), or `ltz` for the Levinson-Trench-Zohar method. The former scales as `O(N log^2 N)` whereas the latter scales as `O(N^2)` and should only be used for `N < 300`.

Value

Vector of n (log-)densities, one for each column of X.

Examples

```
# simulate data
N <- 10 # length of each time series
n <- 3 # number of time series
theta <- 0.1
lambda <- 2
mu <- theta^2 * rep(1, N)
acf <- exp(-lambda * (1:N - 1))

X <- rnormtz(n, acf = acf) + mu

# evaluate log-density
dnormtz(X, mu, acf, log = TRUE)
```

---

fbm_msd *Mean square displacement of fractional Brownian motion.*

Description

Mean square displacement of fractional Brownian motion.

Usage

```
fbm_msd(tseq, H)
```

Arguments

tseq	Length-N vector of timepoints.
H	Hurst parameter (between 0 and 1).

Details

The mean squared displacement (MSD) of a stochastic process X_t is defined as

MSD(t) = E[(X_t - X_0)^2].

Fractional Brownian motion (fBM) is a continuous Gaussian process with stationary increments, such that its covariance function is entirely defined the MSD, which in this case is MSD(t) = |t|^(2H).

Value

Length-N vector of mean square displacements.

Examples

```
fbm_msd(tseq = 1:10, H = 0.4)
```

matern_acf — *Matern autocorrelation function.*

Description

Matern autocorrelation function.

Usage

```
matern_acf(tseq, lambda, nu)
```

Arguments

`tseq`	Vector of `N` time points at which the autocorrelation is to be calculated.
`lambda`	Timescale parameter.
`nu`	Smoothness parameter.

Details

The Matern autocorrelation is given by

$$\mathrm{ACF}(t) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu}\frac{t}{\lambda}\right)^{\nu} K_\nu \left(\sqrt{2\nu}\frac{t}{\lambda}\right),$$

where $K_\nu(x)$ is the modified Bessel function of second kind.

Value

An autocorrelation vector of length `N`.

Examples

```
matern_acf(tseq = 1:10, lambda = 1, nu = 3/2)
```

msd2acf — *Convert mean square displacement of positions to autocorrelation of increments.*

Description

Converts the mean squared displacement (MSD) of a stationary increments sequence `x = (x_0, x_1, ..., x_N)` positions to the autocorrelation of the corresponding increments `dx = (x_1 - x_0, ..., x_N - x_(N-1))`.

Usage

```
msd2acf(msd)
```

Arguments

msd	Length-N MSD vector, i.e., excluding x_0 which is assumed to be zero.

Value

Length-N autocorrelation vector.

Examples

```
# autocorrelation of fBM increments
msd2acf(msd = fbm_msd(tseq = 0:10, H = .3))
```

NormalCirculant *Multivariate normal with Circulant variance matrix.*

Description

Provides methods for the Normal-Circulant (NCt) distribution, which for a random vector z of length N is defined as

```
z ~ NCt(uacf)   <=>   z ~ Normal(0, toeplitz(acf)),
```

where uacf are the Nu = floor(N/2)+1 unique elements of the autocorrelation vector acf, i.e.,

```
acf = (uacf, rev(uacf[2:(Nu-1)]),   N even,
    = (uacf, rev(uacf[2:Nu])),      N odd.
```

Methods

Public methods:

- NormalCirculant$new()
- NormalCirculant$size()
- NormalCirculant$logdens()
- NormalCirculant$grad_full()
- NormalCirculant$clone()

Method new()**:** Class constructor.

Usage:

```
NormalCirculant$new(N)
```

Arguments:

N Size of the NCt random vector.

Returns: A NormalCirculant object.

Method size()**:** Get the size of the NCt random vector.

Usage:

`NormalCirculant$size()`

Returns: Size of the NCt random vector.

Method `logdens()`**:** Log-density function.

Usage:

`NormalCirculant$logdens(z, uacf)`

Arguments:

`z` Density argument. A vector of length `N` or an `N x n_obs` matrix where each column is an `N`-dimensional observation.

`uacf` A vector of length `Nu = floor(N/2)` containing the first half of the autocorrelation (i.e., first row/column) of the Circulant variance matrix.

Returns: A scalar or vector of length `n_obs` containing the log-density of the NCt evaluated at its arguments.

Method `grad_full()`**:** Full gradient of log-density function.

Usage:

`NormalCirculant$grad_full(z, uacf, calc_dldz = TRUE, calc_dldu = TRUE)`

Arguments:

`z` Density argument. A vector of length `N`.

`uacf` A vector of length `Nu = floor(N/2)` containing the first half of the autocorrelation (i.e., first row/column) of the Circulant variance matrix.

`calc_dldz` Whether or not to calculate the gradient with respect to `z`.

`calc_dldu` Whether or not to calculate the gradient with respect to `uacf`.

Returns: A list with elements:

`ldens` The log-density evaluated at `z` and `uacf`.

`dldz` The length-`N` gradient vector with respect to `z`, if `calc_dldz = TRUE`.

`dldu` The length-`Nu = floor(N/2)+1` gradient vector with respect to `uacf`, if `calc_dldu = TRUE`.

Method `clone()`**:** The objects of this class are cloneable with this method.

Usage:

`NormalCirculant$clone(deep = FALSE)`

Arguments:

`deep` Whether to make a deep clone.

NormalToeplitz *Multivariate normal with Toeplitz variance matrix.*

Description

Provides methods for the Normal-Toeplitz (NTz) distribution defined as

```
z ~ NTz(acf)   <=>   z ~ Normal(0, toeplitz(acf)),
```

i.e., for a multivariate normal with mean zero and variance `Tz = toeplitz(acf)`.

Methods

Public methods:

- `NormalToeplitz$new()`
- `NormalToeplitz$size()`
- `NormalToeplitz$logdens()`
- `NormalToeplitz$grad()`
- `NormalToeplitz$hess()`
- `NormalToeplitz$grad_full()`
- `NormalToeplitz$clone()`

Method `new()`: Class constructor.

Usage:

`NormalToeplitz$new(N)`

Arguments:

`N` Size of the NTz random vector.

Returns: A `NormalToeplitz` object.

Method `size()`: Get the size of the NTz random vector.

Usage:

`NormalToeplitz$size()`

Returns: Size of the NTz random vector.

Method `logdens()`: Log-density function.

Usage:

`NormalToeplitz$logdens(z, acf)`

Arguments:

`z` Density argument. A vector of length `N` or an `N x n_obs` matrix where each column is an `N`-dimensional observation.

`acf` A vector of length `N` containing the autocorrelation (i.e., first row/column) of the Toeplitz variance matrix.

Returns: A scalar or vector of length `n_obs` containing the log-density of the NTz evaluated at its arguments.

Method `grad()`**:** Gradient of the log-density with respect to parameters.

Usage:

```
NormalToeplitz$grad(z, dz, acf, dacf, full_out = FALSE)
```

Arguments:

`z` Density argument. A vector of length `N`.

`dz` An `N x n_theta` matrix containing the gradient `dz/dtheta`.

`acf` A vector of length `N` containing the autocorrelation of the Toeplitz variance matrix.

`dacf` An `N x n_theta` matrix containing the gradient `dacf/dtheta`.

`full_out` If `TRUE`, returns the log-density as well (see 'Value').

Returns: A vector of length `n_theta` containing the gradient of the NTz log-density with respect to `theta`, or a list with elements `ldens` and `grad` consisting of the log-density and the gradient vector.

Method `hess()`**:** Hessian of log-density with respect to parameters.

Usage:

```
NormalToeplitz$hess(z, dz, d2z, acf, dacf, d2acf, full_out = FALSE)
```

Arguments:

`z` Density argument. A vector of length `N`.

`dz` An `N x n_theta` matrix containing the gradient `dz/dtheta`.

`d2z` An `N x n_theta x n_theta` array containing the Hessian `d^2z/dtheta^2`.

`acf` A vector of length `N` containing the autocorrelation of the Toeplitz variance matrix.

`dacf` An `N x n_theta` matrix containing the gradient `dacf/dtheta`.

`d2acf` An `N x n_theta x n_theta` array containing the Hessian `dacf^2/dtheta^2`.

`full_out` If `TRUE`, returns the log-density and its gradient as well (see 'Value').

Returns: An `n_theta x n_theta` matrix containing the Hessian of the NTz log-density with respect to `theta`, or a list with elements `ldens`, `grad`, and `hess` consisting of the log-density, its gradient (a vector of size `n_theta`), and the Hessian matrix, respectively.

Method `grad_full()`**:** Full gradient of log-density function.

Usage:

```
NormalToeplitz$grad_full(z, acf, calc_dldz = TRUE, calc_dlda = TRUE)
```

Arguments:

`z` Density argument. A vector of length `N`.

`acf` A vector of length `N` containing the autocorrelation of the Toeplitz variance matrix.

`calc_dldz` Whether or not to calculate the gradient with respect to `z`.

`calc_dlda` Whether or not to calculate the gradient with respect to `acf`.

Returns: A list with elements:

`ldens` The log-density evaluated at `z` and `acf`.

`dldz` The length-`N` gradient vector with respect to `z`, if `calc_dldz = TRUE`.

`dlda` The length-`N` gradient vector with respect to `acf`, if `calc_dlda = TRUE`.

Method `clone()`**:** The objects of this class are cloneable with this method.

Usage:

```
NormalToeplitz$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

`pex_acf` *Power-exponential autocorrelation function.*

Description

Power-exponential autocorrelation function.

Usage

```
pex_acf(tseq, lambda, rho)
```

Arguments

`tseq`	Vector of `N` time points at which the autocorrelation is to be calculated.
`lambda`	Timescale parameter.
`rho`	Power parameter.

Details

The power-exponential autocorrelation function is given by:

$$\mathrm{ACF}(t) = \exp\left\{-(t/\lambda)^\rho\right\}.$$

Value

An autocorrelation vector of length `N`.

Examples

```
pex_acf(tseq = 1:10, lambda = 1, rho = 2)
```

`rnormtz` *Simulate a stationary Gaussian time series.*

Description

Simulate a stationary Gaussian time series.

Usage

```
rnormtz(n = 1, acf, Z, fft = TRUE, nkeep, tol = 1e-06)
```

Arguments

`n`	Number of time series to generate.
`acf`	Length-`N` vector giving the autocorrelation of the series.
`Z`	Optional size `(2N-2) x n` or `N x n` matrix of iid standard normals, to use in the FFT and Durbin-Levinson methods, respectively.
`fft`	Logical; whether or not to use the `O(N log N)` FFT-based algorithm of Wood and Chan (1994) or the more stable `O(N^2)` Durbin-Levinson algorithm. See Details.
`nkeep`	Length of time series. Defaults to `N = length(acf)`. See Details.
`tol`	Relative tolerance on negative eigenvalues. See Details.

Details

The FFT method fails when the embedding circulant matrix is not positive definite. This is typically due to one of two things:

1. Roundoff error can make tiny eigenvalues appear negative. For this purpose, argument `tol` can be used to replace all negative eigenvalues by `tol * ev_max`, where `ev_max` is the largest eigenvalue.
2. The autocorrelation is decaying too slowly on the given timescale. To mitigate this, argument `nkeep` can be used to supply a longer `acf` than is required, and keep only the first `nkeep` time series observations. For consistency, `nkeep` also applies to Durbin-Levinson method.

Value

Length-`nkeep` vector or size `nkeep x n` matrix with time series as columns.

Examples

```
N <- 10
acf <- exp(-(1:N - 1)/N)
rnormtz(n = 3, acf = acf)
```

SuperGauss-defunct *Defunct functions in* **SuperGauss**.

Description

Defunct functions in **SuperGauss**.

The following functions have been removed from the SuperGauss package

`rSnorm()` Please use `rnormtz()` instead.

`dSnorm()` Please use `dnormtz()` instead.

`Snorm.grad()` Please use the `grad()` method in the NormalToeplitz class.

`Snorm.hess()` Please use the `hess()` method in the NormalToeplitz class.

---

toep.mult *Toeplitz matrix multiplication.*

Description

Efficient matrix multiplication with Toeplitz matrix and arbitrary matrix or vector.

Usage

```
toep.mult(acf, X)
```

Arguments

`acf`	Length-`N` vector giving the first column (or row) of the Toeplitz matrix.
`X`	Vector or matrix of compatible dimensions with `acf`.

Value

An `N`-row matrix corresponding to `toeplitz(acf) %*% X`.

Examples

```
N <- 20
d <- 3
acf <- exp(-(1:N))
X <- matrix(rnorm(N*d), N, d)
toep.mult(acf, X)
```

Toeplitz	*Constructor and methods for Toeplitz matrix objects.*

Description

The `Toeplitz` class contains efficient methods for linear algebra with symmetric positive definite (i.e., variance) Toeplitz matrices.

Usage

```
is.Toeplitz(x)

as.Toeplitz(x)

## S3 method for class 'Toeplitz'
dim(x)
```

Arguments

`x`	An R object.

Details

An `N x N` Toeplitz matrix `Tz` is defined by its length-`N` "autocorrelation" vector `acf`, i.e., first row/column `Tz`. Thus, for the function `stats::toeplitz()`, we have `Tz = toeplitz(acf)`.

It is assumed that `acf` defines a valid (i.e., positive definite) variance matrix. The matrix multiplication methods still work when this is not the case but the other methods do not (return values typically contain `NaN`s).

`as.Toeplitz(x)` attempts to convert its argument to a `Toeplitz` object by calling `Toeplitz$new(acf = x)`. `is.Toeplitz(x)` checks whether its argument is a `Toeplitz` object.

Methods

Public methods:

- `Toeplitz$new()`
- `Toeplitz$print()`
- `Toeplitz$size()`
- `Toeplitz$set_acf()`
- `Toeplitz$get_acf()`
- `Toeplitz$has_acf()`
- `Toeplitz$prod()`
- `Toeplitz$solve()`
- `Toeplitz$log_det()`
- `Toeplitz$trace_grad()`
- `Toeplitz$trace_hess()`

- Toeplitz$clone()

Method `new()`**:** Class constructor.

Usage:

`Toeplitz$new(N, acf)`

Arguments:

`N` Size of Toeplitz matrix.

`acf` Autocorrelation vector of length `N`.

Returns: A `Toeplitz` object.

Method `print()`**:** Print method.

Usage:

`Toeplitz$print()`

Method `size()`**:** Get the size of the Toeplitz matrix.

Usage:

`Toeplitz$size()`

Returns: Size of the Toeplitz matrix. `ncol()`, `nrow()`, and `dim()` methods for `Toeplitz` objects also work as expected.

Method `set_acf()`**:** Set the autocorrelation of the Toeplitz matrix.

Usage:

`Toeplitz$set_acf(acf)`

Arguments:

`acf` Autocorrelation vector of length `N`.

Method `get_acf()`**:** Get the autocorrelation of the Toeplitz matrix.

Usage:

`Toeplitz$get_acf()`

Returns: The autocorrelation vector of length `N`.

Method `has_acf()`**:** Check whether the autocorrelation of the Toeplitz matrix has been set.

Usage:

`Toeplitz$has_acf()`

Returns: Logical; `TRUE` if `Toeplitz$set_acf()` has been called.

Method `prod()`**:** Toeplitz matrix-matrix product.

Usage:

`Toeplitz$prod(x)`

Arguments:

`x` Vector or matrix with `N` rows.

Returns: The matrix product `Tz %*% x`. `Tz %*% x` and `x %*% Tz` also work as expected.

Method `solve()`: Solve a Toeplitz system of equations.

Usage:

```
Toeplitz$solve(x, method = c("gschur", "pcg"), tol = 1e-10)
```

Arguments:

`x` Optional vector or matrix with `N` rows.

`method` Solve method to use. Choices are: `gschur` for a modified version of the Generalized Schur algorithm of Ammar & Gragg (1988), or `pcg` for the preconditioned conjugate gradient method of Chen et al (2006). The former is faster and obtains the log-determinant as a direct biproduct. The latter is more numerically stable for long-memory autocorrelations.

`tol` Tolerance level for the `pcg` method.

Returns: The solution in `z` to the system of equations `Tz %*% z = x`. If `x` is missing, returns the inverse of `Tz`. `solve(Tz, x)` and `solve(Tz, x, method, tol)` also work as expected.

Method `log_det()`: Calculate the log-determinant of the Toeplitz matrix.

Usage:

```
Toeplitz$log_det()
```

Returns: The log-determinant `log(det(Tz))`. `determinant(Tz)` also works as expected.

Method `trace_grad()`: Computes the trace-gradient with respect to Toeplitz matrices.

Usage:

```
Toeplitz$trace_grad(acf2)
```

Arguments:

`acf2` Length-`N` autocorrelation vector of the second Toeplitz matrix. This matrix must be symmetric but not necessarily positive definite.

Returns: Computes the trace of

```
solve(Tz, toeplitz(acf2)).
```

This is used in the computation of the gradient of `log(det(Tz(theta)))` with respect to `theta`.

Method `trace_hess()`: Computes the trace-Hessian with respect to Toeplitz matrices.

Usage:

```
Toeplitz$trace_hess(acf2, acf3)
```

Arguments:

`acf2` Length-`N` autocorrelation vector of the second Toeplitz matrix. This matrix must be symmetric but not necessarily positive definite.

`acf3` Length-`N` autocorrelation vector of the third Toeplitz matrix. This matrix must be symmetric but not necessarily positive definite.

Returns: Computes the trace of

```
solve(Tz, toeplitz(acf2)) %*% solve(Tz, toeplitz(acf3)).
```

This is used in the computation of the Hessian of `log(det(Tz(theta)))` with respect to `theta`.

Method `clone()`: The objects of this class are cloneable with this method.

Usage:

```
Toeplitz$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

Examples

```
# construct a Toeplitz matrix
acf <- exp(-(1:5))
Tz <- Toeplitz$new(acf = acf)
# alternatively, can allocate space first
Tz <- Toeplitz$new(N = length(acf))
Tz$set_acf(acf = acf)

# basic methods
Tz$get_acf() # extract the acf
dim(Tz) # == c(nrow(Tz), ncol(Tz))
Tz # print method

# linear algebra methods
X <- matrix(rnorm(10), 5, 2)
Tz %*% X
t(X) %*% Tz
solve(Tz, X)
determinant(Tz) # log-determinant
```

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