

# Package ‘mcauchy’

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mcauchy-package

*Tools for Multivariate Cauchy Distributions*

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## Description

This package provides tools for multivariate Cauchy distributions (MCD):

- Calculation of distances/divergences between MCD:
  - Kullback-Leibler divergence: [kldcauchy](#)
- Tools for MCD:
  - Probability density: [dmcd](#)
  - Simulation from a MCD: [rmcd](#)
  - Plot of the density of a MCD with 2 variables: [plotmcd](#), [contourmcd](#)

## Author(s)

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## References

N. Bouhlef, D. Rousseau, A Generic Formula and Some Special Cases for the Kullback–Leibler Divergence between Central Multivariate Cauchy Distributions. *Entropy*, 24, 838, July 2022. [doi:10.3390/e24060838](https://doi.org/10.3390/e24060838)

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contourmcd

*Contour Plot of the Bivariate Cauchy Density*

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## Description

Draws the contour plot of the probability density of the multivariate Cauchy distribution with 2 variables with location parameter  $\mu$  and scatter matrix  $\Sigma$ .

## Usage

```
contourmcd(mu, Sigma,  
           xlim = c(mu[1] + c(-10, 10)*Sigma[1, 1]),  
           ylim = c(mu[2] + c(-10, 10)*Sigma[2, 2]),  
           zlim = NULL, npt = 30, nx = npt, ny = npt,  
           main = "Multivariate Cauchy density",  
           sub = NULL, nlevels = 10,  
           levels = pretty(zlim, nlevels), tol = 1e-6, ...)
```

**Arguments**

<code>mu</code>	length 2 numeric vector.
<code>Sigma</code>	symmetric, positive-definite square matrix of order 2. The scatter matrix.
<code>xlim, ylim</code>	x-and y- limits.
<code>zlim</code>	z- limits. If NULL, it is the range of the values of the density on the x and y values within <code>xlim</code> and <code>ylim</code> .
<code>npt</code>	number of points for the discretisation.
<code>nx, ny</code>	number of points for the discretisation among the x- and y- axes.
<code>main, sub</code>	main and sub title, as for <a href="#">title</a> .
<code>nlevels, levels</code>	arguments to be passed to the <a href="#">contour</a> function.
<code>tol</code>	tolerance (relative to largest variance) for numerical lack of positive-definiteness in <code>Sigma</code> , for the estimation of the density. see <a href="#">dmcd</a> .
<code>...</code>	additional arguments to <a href="#">plot.window</a> , <a href="#">title</a> , <a href="#">Axis</a> and <a href="#">box</a> , typically <a href="#">graphical parameters</a> such as <code>cex.axis</code> .

**Value**

Returns invisibly the probability density function.

**Author(s)**

Pierre Santagostini, Nizar Bouhlel

**References**

N. Bouhlel, D. Rousseau, A Generic Formula and Some Special Cases for the Kullback–Leibler Divergence between Central Multivariate Cauchy Distributions. *Entropy*, 24, 838, July 2022. [doi:10.3390/e24060838](https://doi.org/10.3390/e24060838)

**See Also**

[dmcd](#): probability density of a multivariate Cauchy density

[plotmcd](#): 3D plot of a bivariate Cauchy density.

**Examples**

```
mu <- c(1, 4)
Sigma <- matrix(c(0.8, 0.2, 0.2, 0.2), nrow = 2)
contourmcd(mu, Sigma)
```

dmcd

*Density of a Multivariate Cauchy Distribution***Description**

Density of the multivariate ( $p$  variables) Cauchy distribution (MCD) with location parameter  $\mu$  and scatter matrix  $\Sigma$ .

**Usage**

```
dmcd(x, mu, Sigma, tol = 1e-6)
```

**Arguments**

<code>x</code>	length $p$ numeric vector.
<code>mu</code>	length $p$ numeric vector. The location parameter.
<code>Sigma</code>	symmetric, positive-definite square matrix of order $p$ . The scatter matrix.
<code>tol</code>	tolerance (relative to largest eigenvalue) for numerical lack of positive-definiteness in <code>Sigma</code> .

**Details**

The density function of a multivariate Cauchy distribution is given by:

$$f(\mathbf{x}|\boldsymbol{\mu}, \Sigma) = \frac{\Gamma\left(\frac{1+p}{2}\right)}{\pi^{p/2}\Gamma\left(\frac{1}{2}\right)|\Sigma|^{\frac{1}{2}}[1 + (\mathbf{x} - \boldsymbol{\mu})^T\Sigma^{-1}(\mathbf{x} - \boldsymbol{\mu})]^{\frac{1+p}{2}}}$$

**Value**

The value of the density.

**Author(s)**

Pierre Santagostini, Nizar Bouhlel

**See Also**

[rmcd](#): random generation from a MCD.  
[plotmcd](#), [contourmcd](#): plot of a bivariate Cauchy density.

**Examples**

```
mu <- c(0, 1, 4)
sigma <- matrix(c(1, 0.6, 0.2, 0.6, 1, 0.3, 0.2, 0.3, 1), nrow = 3)
dmcd(c(0, 1, 4), mu, sigma)
dmcd(c(1, 2, 3), mu, sigma)
```

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kldcauchy	<i>Kullback-Leibler Divergence between Centered Multivariate Cauchy Distributions</i>
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### Description

Computes the Kullback-Leibler divergence between two random vectors distributed according to multivariate Cauchy distributions (MCD) with zero location vector.

### Usage

```
kldcauchy(Sigma1, Sigma2, eps = 1e-06)
```

### Arguments

Sigma1	symmetric, positive-definite matrix. The scatter matrix of the first distribution.
Sigma2	symmetric, positive-definite matrix. The scatter matrix of the second distribution.
eps	numeric. Precision for the computation of the partial derivative of the Lauricella $D$ -hypergeometric function (see Details). Default: 1e-06.

### Details

Given  $X_1$ , a random vector of  $R^p$  distributed according to the MCD with parameters  $(0, \Sigma_1)$  and  $X_2$ , a random vector of  $R^p$  distributed according to the MCD with parameters  $(0, \Sigma_2)$ .

Let  $\lambda_1, \dots, \lambda_p$  the eigenvalues of the square matrix  $\Sigma_1 \Sigma_2^{-1}$  sorted in increasing order:

$$\lambda_1 < \dots < \lambda_{p-1} < \lambda_p$$

Depending on the values of these eigenvalues, the computation of the Kullback-Leibler divergence of  $X_1$  from  $X_2$  is given by:

- if  $\lambda_1 < 1$  and  $\lambda_p > 1$ :

$$KL(X_1||X_2) = -\frac{1}{2} \ln \prod_{i=1}^p \lambda_i + \frac{1+p}{2} \left( \ln \lambda_p - \frac{\partial}{\partial a} \left\{ F_D^{(p)} \left( a, \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_p, a + \frac{1}{2}; a + \frac{1+p}{2}; 1 - \frac{\lambda_1}{\lambda_p}, \dots, 1 - \frac{\lambda_{p-1}}{\lambda_p}, 1 - \frac{1}{\lambda_p} \right) \right\} \Big|_{a=0} \right)$$

- if  $\lambda_p < 1$ :

$$KL(X_1||X_2) = -\frac{1}{2} \ln \prod_{i=1}^p \lambda_i - \frac{1+p}{2} \frac{\partial}{\partial a} \left\{ F_D^{(p)} \left( a, \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_p; a + \frac{1+p}{2}; 1 - \lambda_1, \dots, 1 - \lambda_p \right) \right\} \Big|_{a=0}$$

- if  $\lambda_1 > 1$ :

$$KL(X_1||X_2) = -\frac{1}{2} \ln \prod_{i=1}^p \lambda_i - \frac{1+p}{2} \prod_{i=1}^p \frac{1}{\sqrt{\lambda_i}}$$

$$\times \frac{\partial}{\partial a} \left\{ F_D^{(p)} \left( \frac{1+p}{2}, \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_p; a + \frac{1+p}{2}; 1 - \frac{1}{\lambda_1}, \dots, 1 - \frac{1}{\lambda_p} \right) \right\} \Big|_{a=0}$$

where  $F_D^{(p)}$  is the Lauricella  $D$ -hypergeometric function defined for  $p$  variables:

$$F_D^{(p)}(a; b_1, \dots, b_p; g; x_1, \dots, x_p) = \sum_{m_1 \geq 0} \dots \sum_{m_p \geq 0} \frac{(a)_{m_1 + \dots + m_p} (b_1)_{m_1} \dots (b_p)_{m_p}}{(g)_{m_1 + \dots + m_p}} \frac{x_1^{m_1}}{m_1!} \dots \frac{x_p^{m_p}}{m_p!}$$

The computation of the partial derivative uses the `pochhammer` function.

### Value

A numeric value: the Kullback-Leibler divergence between the two distributions, with two attributes `attr(, "epsilon")` (precision of the partial derivative of the Lauricella  $D$ -hypergeometric function, see Details) and `attr(, "k")` (number of iterations).

### Author(s)

Pierre Santagostini, Nizar Bouhlef

### References

N. Bouhlef, D. Rousseau, A Generic Formula and Some Special Cases for the Kullback–Leibler Divergence between Central Multivariate Cauchy Distributions. *Entropy*, 24, 838, July 2022. doi:10.3390/e24060838

### Examples

```
Sigma1 <- matrix(c(1, 0.6, 0.2, 0.6, 1, 0.3, 0.2, 0.3, 1), nrow = 3)
Sigma2 <- matrix(c(1, 0.3, 0.1, 0.3, 1, 0.4, 0.1, 0.4, 1), nrow = 3)
kldcauchy(Sigma1, Sigma2)
kldcauchy(Sigma2, Sigma1)
```

```
Sigma1 <- matrix(c(0.5, 0, 0, 0, 0.4, 0, 0, 0, 0.3), nrow = 3)
Sigma2 <- diag(1, 3)
# Case when all eigenvalues of Sigma1 %% solve(Sigma2) are < 1
kldcauchy(Sigma1, Sigma2)
# Case when all eigenvalues of Sigma1 %% solve(Sigma2) are > 1
kldcauchy(Sigma2, Sigma1)
```

plotmcd

*Plot of the Bivariate Cauchy Density***Description**

Plots the probability density of the multivariate Cauchy distribution with 2 variables with location parameter  $\mu$  and scatter matrix  $\Sigma$ .

**Usage**

```
plotmcd(mu, Sigma, xlim = c(mu[1] + c(-10, 10)*Sigma[1, 1]),
        ylim = c(mu[2] + c(-10, 10)*Sigma[2, 2]), n = 101,
        xvals = NULL, yvals = NULL, xlab = "x", ylab = "y",
        zlab = "f(x,y)", col = "gray", tol = 1e-6, ...)
```

**Arguments**

<code>mu</code>	length 2 numeric vector.
<code>Sigma</code>	symmetric, positive-definite square matrix of order 2. The scatter matrix.
<code>xlim, ylim</code>	x-and y- limits.
<code>n</code>	A one or two element vector giving the number of steps in the x and y grid, passed to <a href="#">plot3d.function</a> .
<code>xvals, yvals</code>	The values at which to evaluate x and y. If used, <code>xlim</code> and/or <code>ylim</code> are ignored.
<code>xlab, ylab, zlab</code>	The axis labels.
<code>col</code>	The color to use for the plot. See <a href="#">plot3d.function</a> .
<code>tol</code>	tolerance (relative to largest variance) for numerical lack of positive-definiteness in $\Sigma$ , for the estimation of the density. see <a href="#">dmcd</a> .
<code>...</code>	Additional arguments to pass to <a href="#">plot3d.function</a> .

**Value**

Returns invisibly the probability density function.

**Author(s)**

Pierre Santagostini, Nizar Bouhlel

**References**

N. Bouhlel, D. Rousseau, A Generic Formula and Some Special Cases for the Kullback–Leibler Divergence between Central Multivariate Cauchy Distributions. *Entropy*, 24, 838, July 2022. [doi:10.3390/e24060838](https://doi.org/10.3390/e24060838)

**See Also**

[dmcd](#): probability density of a multivariate Cauchy density

[contourmcd](#): contour plot of a bivariate Cauchy density.

[plot3d.function](#): plot a function of two variables.

**Examples**

```
mu <- c(1, 4)
Sigma <- matrix(c(0.8, 0.2, 0.2, 0.2), nrow = 2)
plotmcd(mu, Sigma)
```

---

pochhammer

*Pochhammer Symbol*

---

**Description**

Computes the Pochhammer symbol.

**Usage**

```
pochhammer(x, n)
```

**Arguments**

x	numeric.
n	positive integer.

**Details**

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)\dots(x+n-1)$$

**Value**

Numeric value. The value of the Pochhammer symbol.

**Author(s)**

Pierre Santagostini, Nizar Bouhlel

**Examples**

```
pochhammer(2, 0)
pochhammer(2, 1)
pochhammer(2, 3)
```



---

`rmcd`*Simulate from a Multivariate Cauchy Distribution*

---

**Description**

Produces one or more samples from the multivariate ( $p$  variables) Cauchy distribution (MCD) with location parameter  $\mu$  and scatter matrix  $\Sigma$ .

**Usage**

```
rmcd(n, mu, Sigma, tol = 1e-6)
```

**Arguments**

<code>n</code>	integer. Number of observations.
<code>mu</code>	length $p$ numeric vector. The location parameter.
<code>Sigma</code>	symmetric, positive-definite square matrix of order $p$ . The scatter matrix.
<code>tol</code>	tolerance for numerical lack of positive-definiteness in $\Sigma$ (for <code>mvrnorm</code> , see Details).

**Details**

A sample from a MCD with parameters  $\mu$  and  $\Sigma$  can be generated using:

$$\mathbf{X} = \mu + \frac{\mathbf{Y}}{\sqrt{u}}$$

where  $\mathbf{Y}$  is a random vector distributed among a centered Gaussian density with covariance matrix  $\Sigma$  (generated using `mvrnorm`) and  $u$  is distributed among a Chi-squared distribution with 1 degree of freedom.

**Value**

A matrix with  $p$  columns and  $n$  rows.

**Author(s)**

Pierre Santagostini, Nizar Bouhlel

**See Also**

[dmcd](#): probability density of a MCD.

**Examples**

```
mu <- c(0, 1, 4)
sigma <- matrix(c(1, 0.6, 0.2, 0.6, 1, 0.3, 0.2, 0.3, 1), nrow = 3)
x <- rmcd(100, mu, sigma)
x
apply(x, 2, median)
```

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