PSTricks

pst-math

Special mathematical PostScript functions; v.0.23

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Documentation by
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Contents

`pst-math` is an extension to the PostScript language. The files `pst-math.sty` and `pst-math.tex` are only wrapper files for the `pst-math.pro` file, which defines all the new mathematical functions for use with PostScript.

Thanks to:
Denis Bitouzé; Jacques L'helgoualc'h; Patrice Mégret; Dominik Rodriguez

1 Introduction

`pst-math` defines `\pstPi` on TEX level which expects 1,2,3 or 4 as parameter. It is not available on PostScript level.

`\pstPI#`

`\pstPI1` $\Rightarrow \pi$
`\pstPI2` $\Rightarrow \frac{\pi}{2}$
`\pstPI3` $\Rightarrow \frac{\pi}{3}$
`\pstPI4` $\Rightarrow \frac{\pi}{4}$

2 Trigonometry

`pst-math` introduces natural trigonometric PostScript operators `COS`, `SIN` and `TAN` defined by

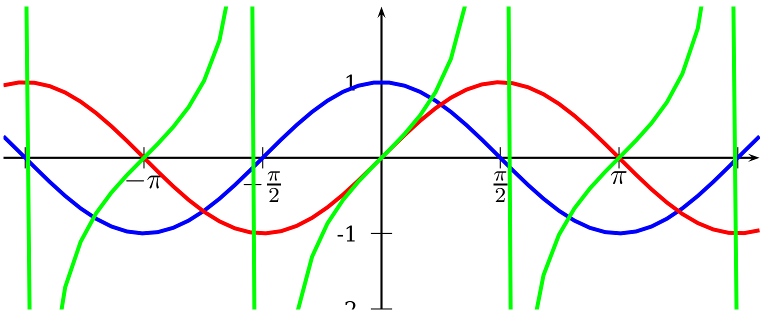
$$\cos : \left\{ \begin{array}{ccc} \mathbb{R} & \rightarrow & [-1,1] \\ x & \mapsto & \cos(x) \end{array} \right.$$

$$\sin : \left\{ \begin{array}{ccc} \mathbb{R} & \rightarrow & [-1,1] \\ x & \mapsto & \sin(x) \end{array} \right.$$

$$\tan : \left\{ \begin{array}{ccc} \mathbb{R}\backslash\{k\frac{\pi}{2}, k \in \mathbb{Z}\} & \rightarrow & \mathbb{R} \\ x & \mapsto & \tan(x) \end{array} \right.$$

where x is in *radians*. `TAN` does *not* produce a PS error[1] when $x = k\frac{\pi}{2}$.

Stack	*Operator*	*Result*	*Description*
num	COS	real	Return cosine of num radians
num	SIN	real	Return sine of num radians
num	TAN	real	Return tangent of num radians




```
\begin{pspicture}*(-5,-2)(5,2)
\SpecialCoor % For label positionning
\psaxes[labels=y,Dx=\pstPI2]{->}(0,0)(-5,-2)(5,2)
```

1 TAN is defined with Div, a special PSTricks operator rather than with div, the default PS operator.

```
\uput[-90](!PI 0){$\pi$} \uput[-90](!PI neg 0){$-\pi$}
\uput[-90](!PI 2 div 0){$\frac{\pi}2$}
\uput[-90](!PI 2 div neg 0){$-\frac{\pi}2$}
\psplot[linewidth=1.5pt,linecolor=blue]{-5}{5}{x COS}
\psplot[linewidth=1.5pt,linecolor=red]{-5}{5}{x SIN}
\psplot[linewidth=1.5pt,linecolor=green]{-5}{5}{x TAN}
\end{pspicture}
```

`pst-math` introduces natural trigonometric postscript operators `ACOS`, `ASIN` and `ATAN` defined by

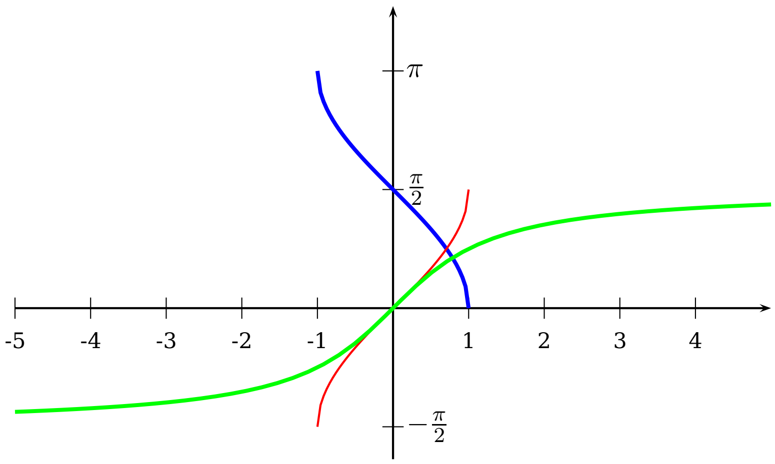
$$\mathrm{acos}: \left\{ \begin{array}{ccc} [-1,1] & \rightarrow & [0,\pi] \\ x & \mapsto & \mathrm{acos}(x) \end{array} \right.$$

$$\mathrm{asin}: \left\{ \begin{array}{ccc} [-1,1] & \rightarrow & [-\frac{\pi}{2},\frac{\pi}{2}] \\ x & \mapsto & \mathrm{asin}(x) \end{array} \right.$$

$$\mathrm{atan}: \left\{ \begin{array}{ccc} \mathbb{R} & \rightarrow &]-\frac{\pi}{2},\frac{\pi}{2}[\\ x & \mapsto & \mathrm{atan}(x) \end{array} \right.$$

Stack	*Operator*	*Result*	*Description*
num	ACOS	angle	Return arccosine of num in radians
num	ASIN	angle	Return arcsine of num in radians
num	ATAN	angle	Return arctangent of num in radians

`ATAN` is *not* defined as the already existing PS operator `atan`. `ATAN` needs only *one* argument on the stack.




```
\begin{pspicture}(-5,-2)(5,4)
\SpecialCoor % For label positionning
\psaxes[labels=x,Dy=\pstPI2]{->}(0,0)(-5,-2)(5,4)
\uput[0](!0 PI){$\pi$} \uput[0](!0 PI 2 div){$\frac{\pi}2$}
\uput[0](!0 PI 2 div neg){$-\frac{\pi}2$}
\psplot[linewidth=1.5pt,linecolor=blue]{-1}{1}{x ACOS} \psplot[linecolor=red
  ]{-1}{1}{x ASIN}
\psplot[linewidth=1.5pt,linecolor=green]{-5}{5}{x ATAN}
\end{pspicture}
```

3 Hyperbolic trigonometry

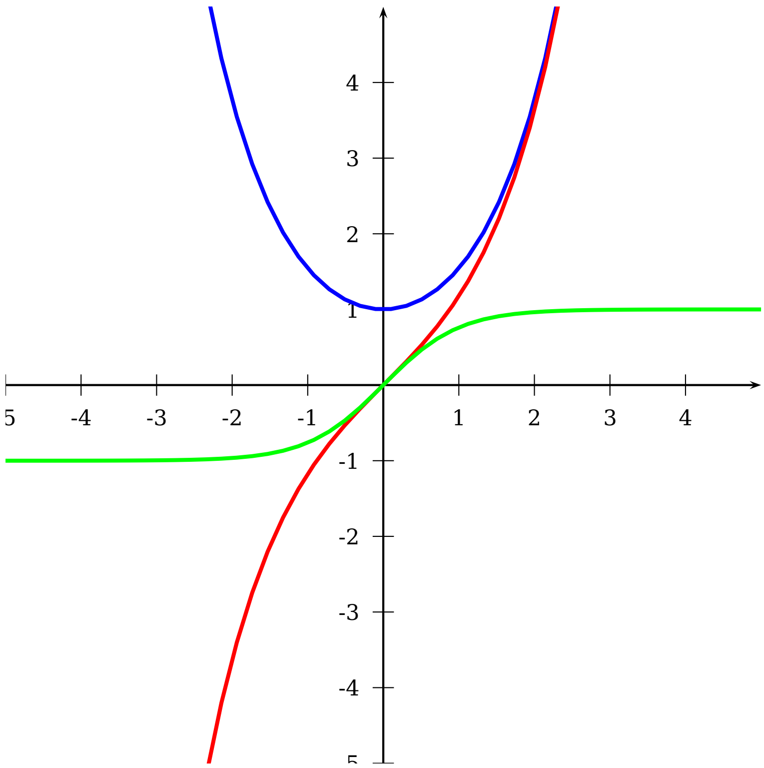
`pst-math` introduces hyperbolic trigonometric postscript operators `COSH`, `SINH` and `TANH` defined by

$$\cosh : \left\{ \begin{array}{ccc} \mathbb{R} & \longrightarrow & [1, +\infty[\\ x & \longmapsto & \cosh(x) \end{array} \right.$$

$$\sinh : \left\{ \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & \sinh(x) \end{array} \right.$$

$$\tanh : \left\{ \begin{array}{ccc} \mathbb{R} & \longrightarrow &]-1, 1[\\ x & \longmapsto & \tanh(x) \end{array} \right.$$

Stack	*Operator*	*Result*	*Description*
num	`COSH`	real	Return hyperbolic cosine of num
num	`SINH`	real	Return hyperbolic sine of num
num	`TANH`	real	Return hyperbolic tangent of num




```
\begin{pspicture}*(-5,-5)(5,5)
\psaxes{->}(0,0)(-5,-5)(5,5)
\psplot[linewidth=1.5pt,linecolor=blue]{-5}{5}{x COSH}
\psplot[linewidth=1.5pt,linecolor=red]{-5}{5}{x SINH}
\psplot[linewidth=1.5pt,linecolor=green]{-5}{5}{x TANH}
\end{pspicture}
```

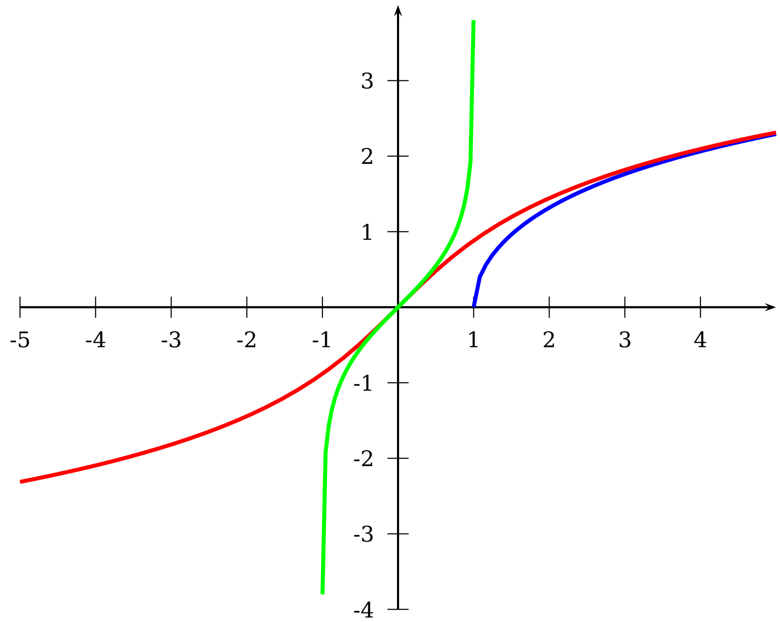
pst-math introduces reciprocal hyperbolic trigonometric postscript operators ACOSH, ASINH and ATANH defined by

$$\text{acosh} : \left\{ \begin{array}{ccc} [1,+\infty[& \longrightarrow & \mathbb{R} \\ x & \longmapsto & \text{acosh}(x) \end{array} \right.$$

$$\text{asinh} : \left\{ \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & \text{asinh}(x) \end{array} \right.$$

$$\text{atanh} : \left\{ \begin{array}{ccc}]-1,1[& \longrightarrow & \mathbb{R} \\ x & \longmapsto & \text{atanh}(x) \end{array} \right.$$

Stack	*Operator*	*Result*	*Description*
num	ACOSH	real	Return reciprocal hyperbolic cosine of num
num	ASINH	real	Return reciprocal hyperbolic sine of num
num	ATANH	real	Return reciprocal hyperbolic tangent of num



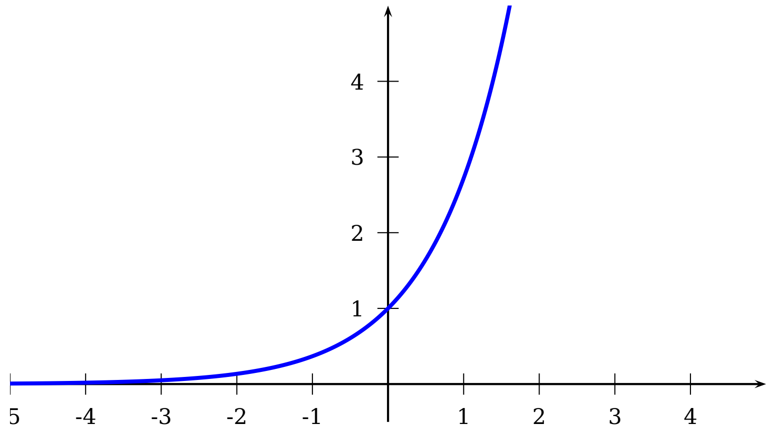

```
\begin{pspicture}(-5,-4)(5,4)
\psaxes{->}(0,0)(-5,-4)(5,4)
\psplot[linewidth=1.5pt,linecolor=blue]{1}{5}{x ACOSH}
\psplot[linewidth=1.5pt,linecolor=red]{-5}{5}{x ASINH}
\psplot[linewidth=1.5pt,linecolor=green]{-.999}{.999}{x ATANH}
\end{pspicture}
```

4 Other operators

pst-math introduces postscript operator EXP defined by

$$\exp : \left\{ \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & \exp(x) \end{array} \right.$$

Stack	*Operator*	*Result*	*Description*
num	EXP	real	Return exponential of num

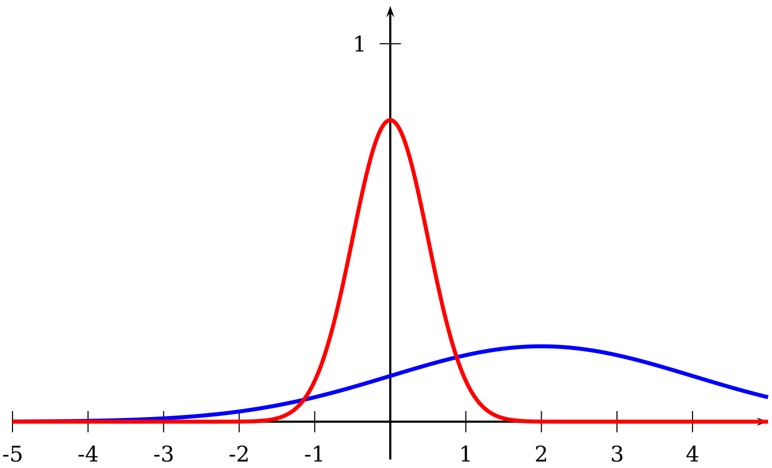



```
\begin{pspicture}*(-5,-1)(5,5)
\psaxes{->}(0,0)(-5,-0.5)(5,5)
\psplot[linecolor=blue,linewidth=1.5pt,plotpoints=1000]{-5}{5}{x EXP}
\end{pspicture}
```

`pst-math` introduces postscript operator `GAUSS` defined by

$$\text{gauss} : \left\{ \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & \dfrac{1}{\sqrt{2\pi\sigma^2}} \exp - \dfrac{(x-\overline{x})^2}{2\sigma^2} \end{array} \right.$$

Stack	*Operator*	*Result*	*Description*
num1 num2 num3	GAUSS	real	Return gaussian of num1 with mean num2 and standard deviation num3

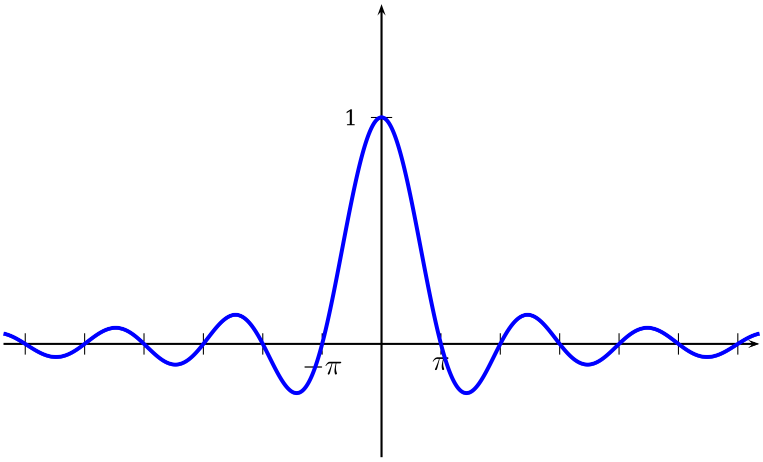



```
\psset{yunit=5}
\begin{pspicture}(-5,-.1)(5,1.1)
\psaxes{->}(0,0)(-5,-.1)(5,1.1)
\psplot[linecolor=blue,linewidth=1.5pt,plotpoints=1000]{-5}{5}{x 2 2 GAUSS}
\psplot[linecolor=red,linewidth=1.5pt,plotpoints=1000]{-5}{5}{x 0 .5 GAUSS}
\end{pspicture}
```

`pst-math` introduces postscript operator SINC defined by

$$\operatorname{sinc}:\left\{\begin{array}{ccc}\mathbb{R} & \longrightarrow & \mathbb{R}\\ x & \longmapsto & \dfrac{\sin x}{x}\end{array}\right.$$

Stack	*Operator*	*Result*	*Description*
num	SINC	real	Return cardinal sine of num radians



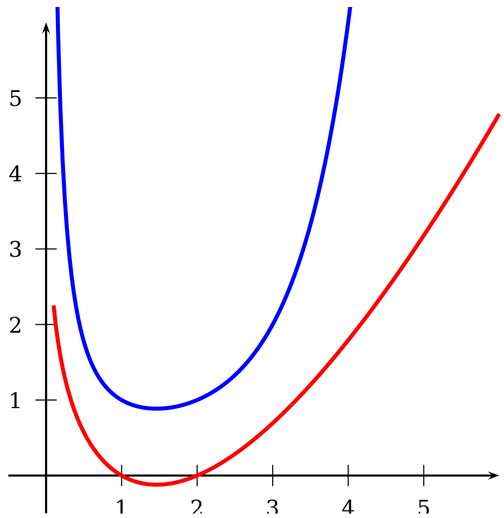

```
\psset{xunit=.25,yunit=3}
\begin{pspicture}(-20,-.5)(20,1.5)
\SpecialCoor % For label positionning
\psaxes[labels=y,Dx=\pstPI1]{->}(0,0)(-20,-.5)(20,1.5)
\uput[-90](!PI 0){$\pi$} \uput[-90](!PI neg 0){$-\pi$}
\psplot[linecolor=blue,linewidth=1.5pt,plotpoints=1000]{-20}{20}{x SINC}
\end{pspicture}
```

`pst-math` introduces postscript operator `GAMMA` and `GAMMALN` defined by

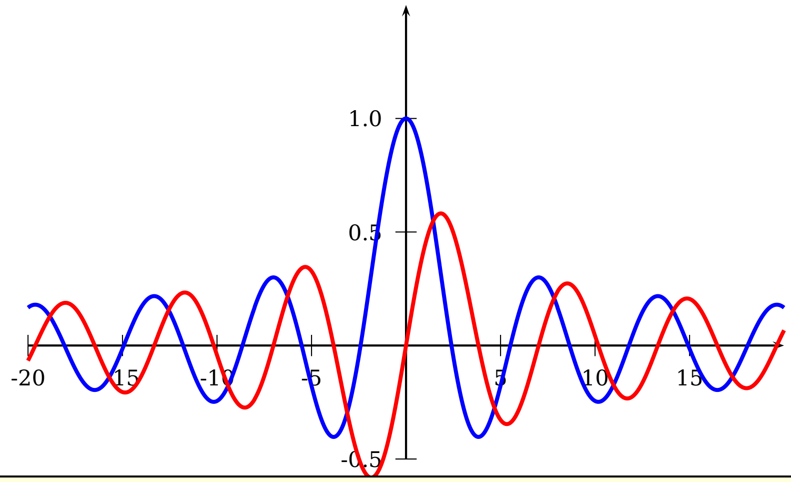
$$\Gamma:\left\{\begin{array}{ccc}\mathbb{R}\backslash\mathbb{Z} & \longrightarrow & \mathbb{R}\\ x & \longmapsto & \displaystyle\int_0^{\infty} t^{x-1}\mathrm{e}^{-t}\,\mathrm{d}t\end{array}\right.$$

$$\ln\Gamma:\left\{\begin{array}{ccc}]0,+\infty[& \longrightarrow & \mathbb{R}\\ x & \longmapsto & \ln\displaystyle\int_0^{t} t^{x-1}\mathrm{e}^{-t}\,\mathrm{d}t\end{array}\right.$$

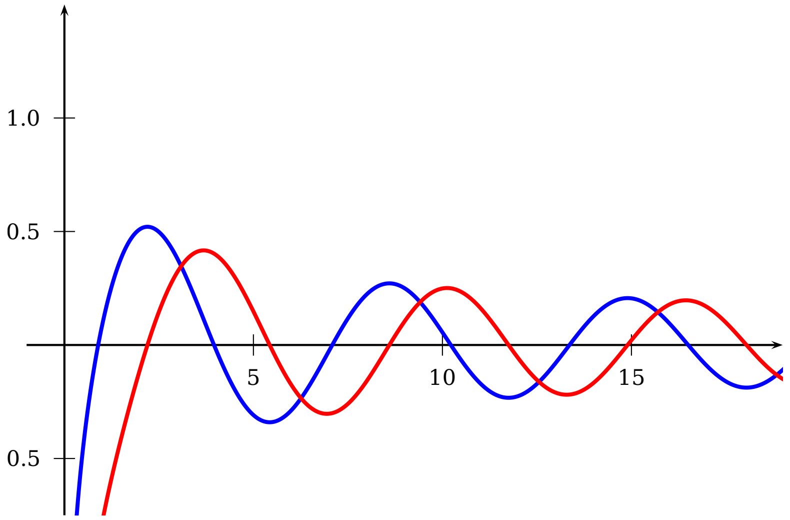
Stack	*Operator*	*Result*	*Description*
num	GAMMA	real	Return Γ function of num
num	GAMMALN	real	Return logarithm of Γ function of num




```
\begin{pspicture*}(-.5,-.5)(6.2,6.2)
\psaxes{->}(0,0)(-.5,-.5)(6,6)
\psplot[linecolor=blue,linewidth=1.5pt,plotpoints=200]{.1}{6}{x GAMMA}
\psplot[linecolor=red,linewidth=1.5pt,plotpoints=200]{.1}{6}{x GAMMALN}
\end{pspicture*}
```




```
\psset{xunit=.25,yunit=3}
\begin{pspicture}(-20,-.5)(20,1.5)
\psaxes[Dx=5,Dy=.5]{->}(0,0)(-20,-.5)(20,1.5)
\psplot[linecolor=blue,linewidth=1.5pt,plotpoints=1000]{-20}{20}{x BESSEL_J0}
\psplot[linecolor=red,linewidth=1.5pt,plotpoints=1000]{-20}{20}{x BESSEL_J1}
\end{pspicture}
```

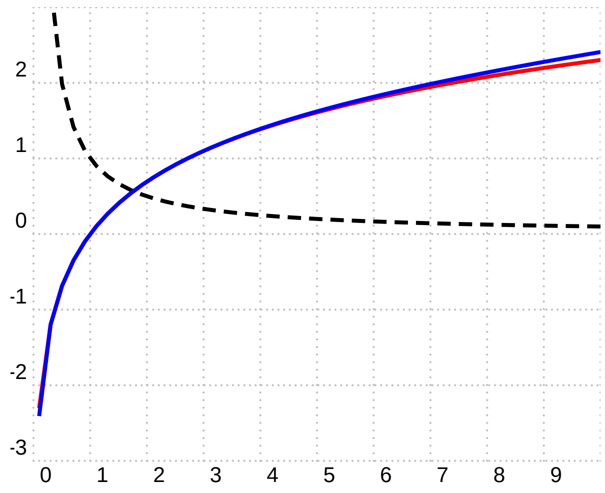



```
\psset{xunit=.5,yunit=3}
\begin{pspicture}*(-1.5,-.75)(19,1.5)
\psaxes[Dx=5,Dy=.5]{->}(0,0)(-1,-.75)(19,1.5)
\psplot[linecolor=blue,linewidth=1.5pt,plotpoints=1000]{0.0001}{20}{x BESSEL_Y0}
\psplot[linecolor=red,linewidth=1.5pt,plotpoints=1000]{0.0001}{20}{x BESSEL_Y1}
%\psplot[linecolor=green,plotpoints=1000]{0.0001}{20}{x 2 BESSEL_Yn}
\end{pspicture}
```

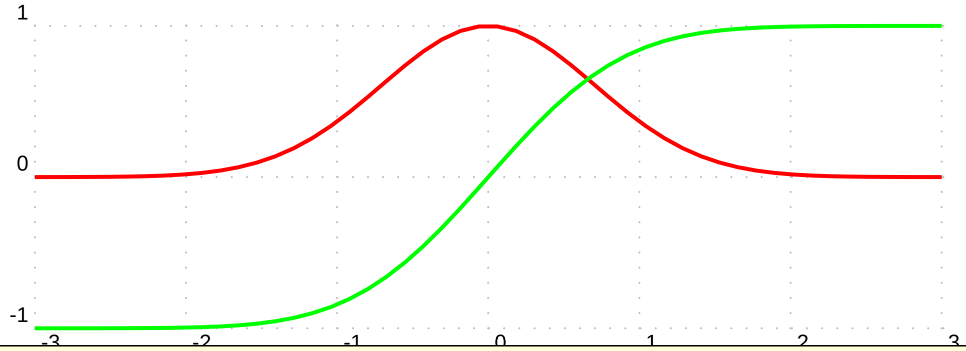
5 Numerical integration

Stack	*Operator*	*Result*	*Description*
num num /var { function } num	SIMPSON	real	Return $\int\limits_a^b f(t)\mathrm{d}t$

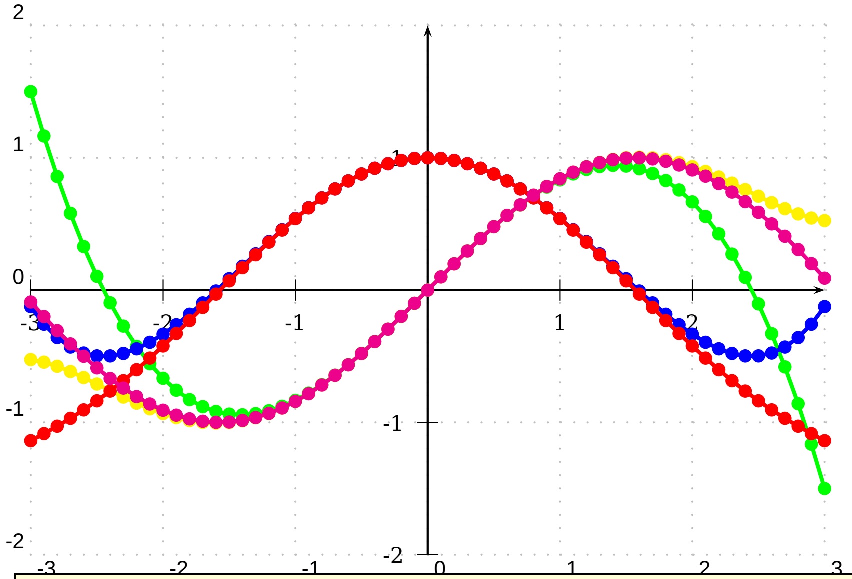
the first two variables are the low and high boundary integral, both can be values or `PostScript` expressions. `/var` is the definition of the integrated variable (not x!), which is used in the following function description, which must be inside of braces. The last number is the tolerance for the step adjustment. The function `SIMPSON` can be nested.




```
\psset{xunit=.75}
\begin{pspicture*}[showgrid=true](-0.4,-3.4)(10,3)
  \psplot[linestyle=dashed,linewidth=1.5pt]{.1}{10}{1 x div}
  \psplot[linecolor=red,linewidth=1.5pt]{.1}{10}{
    1        % start
    x        % end
    /t       % variable
    { 1 t div } % function
    .001     % tolerance
    SIMPSON } %
  \psplot[linecolor=blue,linewidth=1.5pt]{.1}{10}{1 x /t { 1 t div } 1 SIMPSON }
\end{pspicture*}
```

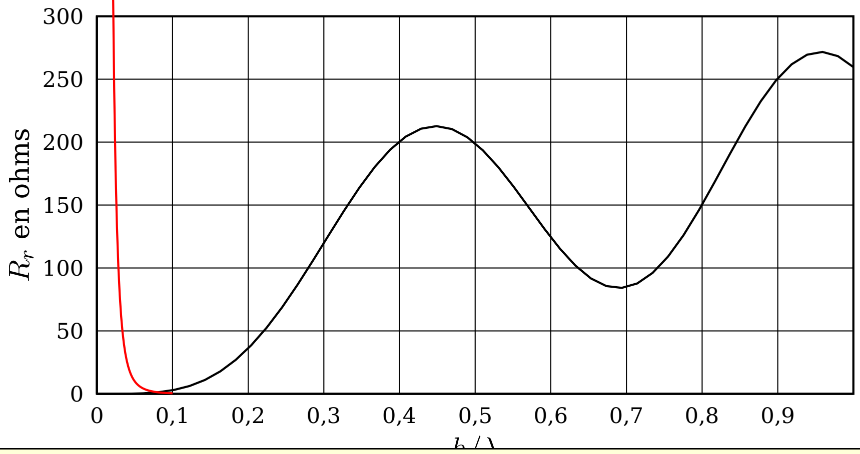



```
%%% Gaussian and relative integral from -x to x to its value sqrt{pi}
\psset{unit=2}
\begin{pspicture}[showgrid=true](-3,-1)(3,1)
  \psplot[linecolor=red,linewidth=1.5pt]{-3}{3}{Euler x dup mul neg exp }
  \psplot[linecolor=green,linewidth=1.5pt]{-3}{3}
    { x neg x /t { Euler t dup mul neg exp } .001 SIMPSON Pi sqrt div}
\end{pspicture}
```




```
\psset{unit=1.75cm}
%%% successive polynomial developments of sine-cosine
\begin{pspicture}[showgrid=true](-3,-2)(3,2)
\psaxes{->}(0,0)(-3,-2)(3,2)
 \psplot[linecolor=green, algebraic=false, plotpoints=61, showpoints=true]
      {-3}{3}{0 x /tutu
            {1 0 tutu /toto { toto } .1 SIMPSON sub}
              .01 SIMPSON }
 \psplot[linecolor=blue, algebraic=false, plotpoints=61, showpoints=true]
      {-3}{3}{1 0 x /tata
            {0 tata /tutu
             {1 0 tutu /toto { toto } .1 SIMPSON sub}
              .01 SIMPSON }
              .01 SIMPSON sub}
 \psplot[linecolor=yellow, algebraic=false, plotpoints=61, showpoints=true]
      {-3}{3}{0 x /titi
            {1 0 titi /tata
             {0 tata /tutu
              {1 0 tutu /toto { toto } .1 SIMPSON sub}
               .01 SIMPSON }
                .01 SIMPSON sub}
                 .01 SIMPSON }
 \psplot[linecolor=red, algebraic=false, plotpoints=61, showpoints=true]
      {-3}{3}{1 0 x /tyty
            {0 tyty /titi
             {1 0 titi /tata
              {0 tata /tutu
               {1 0 tutu /toto { toto } .1 SIMPSON sub}
                .01 SIMPSON }
                 .01 SIMPSON sub}
                  .01 SIMPSON }
                   .01 SIMPSON sub}
 \psplot[linecolor=magenta, algebraic=false, plotpoints=61, showpoints=true]
      {-3}{3}{0 x /tete
            {1 0 tete /tyty
             {0 tyty /titi
              {1 0 titi /tata
               {0 tata /tutu
                {1 0 tutu /toto { toto } .1 SIMPSON sub}
```

```
                 .01 SIMPSON }
                  .01 SIMPSON sub}
                   .01 SIMPSON }
                    .01 SIMPSON sub}
                     .01 SIMPSON }%%% FIVE nested calls
\end{pspicture}
```




```
% ce code definit la fonction [cos(2pix cos(t))-cos(2pix)]^2 / sin(t) avec x=h/
  lambda
\def\F{
 0.01 3.1
 /t
 { TwoPi x mul t COS mul COS TwoPi x mul COS sub 2 exp t SIN div }
  .01 SIMPSON 60 mul }
% D = 2*(cos^2(2pix))/F
\def\fD{TwoPi x mul COS dup mul 2 mul \F\space div}
\psset{llx=-1.5cm,lly=-0.5cm,urx=0.2cm,ury=0.2cm,
 xAxisLabel={$h/\lambda$},xAxisLabelPos={0.5,-45},yAxisLabel={$R_r$ en ohms},
 yAxisLabelPos={-0.1,150}}
\begin{psgraph}[Dy=50,Dx=0.1,xticksize=300 0,yticksize=1 0,
 comma=true,axesstyle=frame](0,0)(1,300){10cm}{5cm}
 \psplot{0}{1}{\F}
 \psplot[linecolor=red]{0.01}{.1}{\fD}%
\end{psgraph}
```

References

[1] Denis Girou. Présentation de PSTricks. *Cahier GUTenberg*, 16:21–70, April 1994.

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[11] Timothy van Zandt. *pst-plot: Plotting two dimensional functions and data*. CTAN:graphics/pstricks/generic/pst-plot.tex, 1999.

[12] Timothy van Zandt and Denis Girou. Inside PSTricks. *TUGboat*, 15:239–246, September 1994.

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